Measurement uncertainty of quantity modelled with non-integer order differ-integral equations

Abstract

Real-time systems require controlled operation to maintain the desired performance, typically ensured by feedback controllers. Control design algorithms rely on accurate modeling techniques that describe the dynamic behavior of physical systems. The developed measurement models capture system dynamics based on the observed input-output data.

Dynamic measurement systems often exhibit distributed dynamics and nonlinear behavior. Sensory measurements capture the transient and steady-state responses in such systems. These dynamic properties arise from different physical phenomena and distinct uncertainty sources. These factors influence both modeling accuracy and controller performance.

Classical methods address uncertainty in static measurements and Integer-Order (IO) dynamic models. However, methods for non-integer-order differ-integral equations remain insufficiently developed. This gap limits the application of Fractional-Order (FO) modelling in metrology, despite its proven capability to represent complex physical dynamics with high accuracy.

This dissertation examines uncertainty in quantities described by FO dynamic equations. It integrates metrology, model identification, fractional calculus, and control theory. The work establishes a framework for uncertainty evaluation in systems exhibiting FO behavior.

The principal contribution of this dissertation is the evaluation of uncertainty propagated through the identified parameters of FO models. It develops a direct uncertainty propagation framework based on time-dependent sensitivity coefficients derived analytically from FO differ-integral equations. The proposed method incorporates the covariance of the identified parameters and applies the law of propagation of uncertainty to quantify the uncertainty associated with the identified parameters. Experimental data from several measurement passes indicate that the estimated parameters follow normal probability distributions, which enables a probabilistic description. The resulting uncertainty intervals calculated from the time-dependent sensitivity coefficients and the covariance matrix of the the normally distributed parameters enclose the measured data. This experimental analysis confirms the validity of the proposed method. Comparison with digital filtering methods indicates that the proposed approach yields conservative uncertainty estimates and avoids information loss associated with signal transformation.

The methodology subsequently extends the uncertainty evaluation to closed-loop systems. It derives sensitivity coefficients of the closed-loop FO model and utilizes these coefficients to assess uncertainty propagated through controlled dynamics. Finally, it conducts a comparative analysis of uncertainty propagation in the closed-loop response using GrünwaldLetnikov, Riemann-Liouville, and Caputo implementations of FO operators. The experimental results demonstrate that the FO differential integral equations can be integrated into a rigorous metrological framework for dynamic uncertainty evaluation.