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## **Review of the Doctoral Dissertation**

of

**Faisal Saleem, M.Sc. Eng.**

**Dissertation Title:** *"Measurement uncertainty of quantity modelled with non-integer order differ-integral equations"*

**Field:** Engineering and Technical Sciences ,

**Discipline:** Automatic Control, Electronics, Electrical Engineering and Space Technologies.

The reviewed dissertation was prepared under the scientific supervision of **Józef Wiora, D.Sc. Eng., Professor of the Silesian University of Technology**, and **Alicja Wiora, Ph.D. Eng.** (auxiliary supervisor). This review was prepared based on Resolution No. RDAEETK.512.17.2026 of the Council of the AEEiTK Discipline of the Silesian University of Technology, dated 31.07.2026.

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### **1 Scope and Purpose of the Dissertation**

Currently, both in science and industry, there is a growing demand for the development of new methods and techniques for digital modelling of many real physical systems. Properly developed and validated digital models are essential for building "digital twins," processing measurement signals, or designing control systems.

The current state of knowledge also indicates that models based on non-integer (fractional) order differential calculus are being successfully applied in digital modelling. "Fractional" models are in many situations more accurate than their integer-order counterparts; however, many issues in this area still await solutions.

One of the key stages in building a model of any real process or object is the identification of its parameters based on data obtained from measurements, which by their nature are subject to uncertainty. Analysis of model accuracy, implied by the uncertainty of measurement data, is crucial for determining its correctness and usefulness.

Known methods of uncertainty analysis concern steady states and (in the case of transient states) systems described by integer-order differential equations. There is a lack of a developed research methodology allowing for comprehensive analysis of static and dynamic accuracy of models described by non-integer order equations.

The **aim and subject** of the reviewed dissertation is the analysis of uncertainty of models described by fractional models. The work integrates issues from the fields of metrology, model identification, fractional calculus, and control theory. Within its

framework, a methodology for assessing the uncertainty of non-integer order models has been developed and validated.

**Research Hypotheses (p. 4, free translation):**

- **H1.** The fractional approach allows for an accurate description of complex nonlinear systems. A fractional model requires fewer parameters than classical integer-order models.
- **H2.** In the case of fractional models built using the Grünwald–Letnikov (GL) definition, memory length has a significant impact on measurement uncertainty and model accuracy. In particular, it is possible to select a memory length ensuring a given model accuracy.
- **H3.** The uncertainty component related to identified parameters can be separated and quantified in the output space without the need for intermediate transformations. This significantly simplifies the analysis and interpretation of results.

The first of the above hypotheses is obvious, since currently one of the main applications of fractional calculus is modelling. The remaining two, relating to uncertainty analysis, are formulated accurately and correctly describe existing research gaps that the reviewed dissertation aims to fill.

**Research Tasks (p. 3):**

- **Z1.** Analysis of the impact of a selected approximation on model uncertainty.
- **Z2.** Development of a mathematical model describing the dynamic response of a laboratory flow measurement system.
- **Z3.** Development of a non-integer order model of the flow process using a difference equation based on the GL definition.
- **Z4.** Analysis of the quality and robustness of the fractional flow model and comparison with integer-order models.
- **Z5.** Proposing tuning rules for the FOPI controller and analysis of the impact of applied approximation methods for the non-integer order operator on control quality in the considered cases.
- **Z6.** Development of a research methodology enabling dynamic modelling of uncertainty propagation using time-dependent sensitivity coefficients.

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**2 Overview of the Dissertation Content**

The dissertation text totals **221 pages** and is divided into **9 main chapters**, a bibliography, a list of abbreviations, lists of figures and tables, and appendices. An overview of the content is discussed below.

The dissertation begins with **abstracts in English and Polish**.

Next, a **list of the Author's publications** is provided, divided into a part directly related to the dissertation and other publications. In the part directly related to the dissertation, there are 3 co-authored articles in journals worth 200, 140, and 100 points, in 2 of which the Author is the first author, plus 4 co-authored conference papers. In the unrelated part, 5 articles of varying scoring are listed.

Next comes the **table of contents**, and lists of figures, tables, and acronyms.

## ■ Chapter 1 - Introduction

Identifies the existing research gap, the essence of which is the lack of a developed methodology for dynamic uncertainty analysis of systems described by non-integer order models, identified based on noisy data collected with variable sampling periods. The identified gap allowed for formulating the **objectives** of the work: developing a unified framework for dynamic measurement uncertainty analysis in systems described by non-integer order equations, and, on this basis, developing tuning rules for a fractional PI controller (FOPI) with an approximated integrating part.

The final part of this chapter discusses the content of the dissertation, with a reference of individual tasks to individual chapters (Table 1.1).

## ■ Chapter 2 - Theoretical Background and State of the Art

It contains basic theoretical background and an extensive "state of the art" analysis in the scope of the dissertation. It begins with basic concepts from metrology, necessary for presenting the main achievements of the work.

It then presents basic concepts of non-integer order calculus, necessary for describing and presenting the main results of the dissertation.

Next, known approaches to modelling — both integer and non-integer order — are discussed, along with numerous examples of models of real processes and phenomena.

Further on, measurement models accounting for dynamics and the description of uncertainty and disturbances of measured quantities are discussed.

Methods for designing closed-loop control systems with PID controllers (both integer and non-integer order) are then discussed, with particular emphasis on the impact of applied approximations on control quality in the system.

The final part of this chapter discusses the limitations and challenges posed to the considered models by measurement uncertainty and disturbances.

## ■ Chapter 3 - Considered Dynamic Systems

It presents the dynamic systems applied to generate experimental data and validate the proposed solutions. For this purpose, the following were used: two fractional-order inertial transfer functions without delay, modelled in MATLAB/SIMULINK; a laboratory helicopter model (TRAS system); and a laboratory flow measurement system.

As examples of real systems described by fractional-order transfer functions, systems with elastically coupled rotating masses are given, and the issue of time units describing pseudo time constants in models, still a debatable issue when applying fractional models is discussed.

For the TRAS system, its nonlinear model, its linearization around the equilibrium point, and the operator transfer function matrix are given. Numerical values of parameters describing the real laboratory model are also provided for the considered models.

For the laboratory flow system model, detailed hardware schematics, software, measurement algorithms and their implementation in MATLAB, and example measurement results are provided.

#### ■ **Chapter 4 - Approximation Accuracy**

It discusses the accuracy of approximations used in non-integer order modelling and control. This issue is one of the fundamental ones in the considered dissertation.

First, the accuracy of approximated fractional transfer functions is discussed. Definitions of errors analyzed further in the dissertation are given first: static errors in the time and frequency domains, as well as the IAE quality index and the  $H^\infty$  norm used to estimate the accuracy of transient responses.

Formulas describing steady-state errors for all transfer functions approximating fractional models considered in the dissertation are then given.

Next, step response plots of fractional transfer functions for various orders and considered approximation methods are presented.

A very thorough numerical analysis of accuracy in terms of Nyquist frequency characteristics is then discussed and presented, measured by gain and phase errors, with the exact model's gain and phase values taken as reference.

Model accuracy analysis is then performed.

#### ■ **Chapter 5 - Integer-Order Flow Process Models**

It describes integer-order models of the flow process in the form of 2nd and 3rd order transfer functions. The method of identifying model parameters using the MAE cost function and the method of obtaining a reduced model are described. Accuracy analysis of the proposed models is then performed.

#### ■ **Chapter 6 - Proposed Non-Integer Order Models**

It presents the non-integer order models proposed by the Author for the liquid flow process. The proposed model is based on the Grünwald–Letnikov definition, and its parameters are identified based on the step response using a framework built by the Author.

The initial part of this chapter includes a "state of the art" review in the field of fractional flow modelling.

A model of flow processes is then proposed in the form of a simple fractional-order transfer function with one zero and one pole. The proposed model has 5 parameters requiring identification and is designed to describe the behavior of the modelled process's response, but its parameters have no physical interpretation.

The proposed fractional model was then compared with integer-order models: a 3rd order model without delay and a 2nd order model with delay.

For digital implementation, an AMPL model was built, which was subsequently used for identification using the NEOS server.

The identification of parameters for all models (integer and non-integer order) based on experimental data is then discussed, and a thorough analysis of their accuracy, robustness, sensitivity to parameter disturbances, and numerical errors during their use is performed.

An analysis of the impact of the applied solvers on the accuracy of the obtained results and computation times during optimization and step response determination was also carried out.

## ■ Chapter 7 - FOPI Controller Design

It is devoted to the design of a control system with a FOPI controller for the TRAS system. An analytical-numerical method for determining controller parameters is proposed, based on given frequency properties of the closed-loop system, and the control quality is then compared with that obtained from an integer-order PI controller tuned using the pidtune tool available in MATLAB.

An analysis of control quality and robustness depending on the applied approximation method for the non-integer order operator was also carried out, considering ORA, Charef, and Matsuda approximations.

## ■ Chapter 8 - Dynamic Uncertainty Propagation

It discusses the propagation of dynamic measurement uncertainty in fractional-order models describing liquid flow. To describe it, a dynamic model of the considered process was built using the GL definition, and an analytical formula for determining the model's response to any input was given. The parameters of this model were determined based on experimental data by minimizing the Mean Square Error (MSE) cost function describing the difference between the model's step response and that of the real object.

The obtained model was then used to determine the propagation of measurement uncertainty arising during numerical identification of model parameters. This is measured by the proposed dynamic sensitivity indices of each model parameter to the uncertainty of experimental data, which were derived analytically (in ready-to-apply form) and whose properties were theoretically justified.

Further on, error and uncertainty propagation in a closed-loop control system containing a fractional-order object with delay and a FOPI controller was considered. In this case, dynamic sensitivity indices of the control system to the uncertainty of the object model's parameters were also proposed.

Dynamic sensitivity indices of the control system were then presented for the situation where the non-integer order operator is described by the Riemann–Liouville or Caputo definitions. The application of the same definitions to describe both the controller and the object, or describing the object and controller using different definitions, was considered.

## ■ Chapter 9 - Summary and Future Research

It constitutes a summary and presentation of plans for further research on the considered topic. At the beginning, the content of all previous chapters is recalled and the most important scientific achievements of the dissertation are summarized.

Next, the **limitations** of the proposed methods are discussed, namely:

- ☒ High computational complexity resulting from the need to calculate GL definition coefficients, while lacking the possibility of adaptively determining the order of the definition,
- ☒ Analysis limited to a SISO system whose parameters are described by a normal distribution — there is no analysis for the situation where parameters are described by another distribution,
- ☒ In the dissertation, only the propagation of measurement uncertainty related to identified model parameters is analyzed — no analysis of uncertainty originating from other sources was performed,
- ☒ Uncertainty propagation in systems described by non-integer order differential equations was not analyzed.

It should be emphasized here that the above critical analysis of the presented achievements speaks very well of the scientific maturity of the Author of the dissertation.

**Directions for further research** indicated include, among others:

- ☒ Considering the order of approximation using the GL definition as an additional decision variable in identification, in order to reduce computational complexity,
- ☒ Application of Bayesian approximation for identification,
- ☒ Considering other sources of uncertainty in measurement data.

**Appendix A** presents the Microsoft Copilot prompt used for linguistic correction of the dissertation text in the form of LaTeX source code.

At the end of the work is a very extensive **bibliography**, comprising **184 items**, selected accurately and fully confirming the good grounding of the dissertation's subject matter in the current state of research.

### **3 Achievements of the Dissertation**

The achievements of the dissertation are briefly described by the Author on p. 4 (C1–C5); however, preparing the review requires identifying them based on content analysis.

Accordingly, I state that the **main achievements** of the dissertation are discussed in Chapters 4–8. In particular, they are as follows:

**(O1)** A very thorough comparative analysis of the accuracy of various methods for approximating fractional-order transfer functions in terms of frequency characteristics.

**(O2)** Proposing a new non-integer order model describing the liquid flow process, its identification, and a thorough analysis of its accuracy, sensitivity to parameter uncertainty, and numerical errors.

**(O3)** Development of an analytical-numerical method for tuning a FOPI controller for a nonlinear system described by an approximate linear model, and analysis of the impact of the selected approximation method on control quality in the system.

**(O4)** Proposing dynamic sensitivity indices of the model and control system to the uncertainty of experimental data, deriving analytical formulas for these coefficients, and their theoretical analysis. The object model takes the form of a fractional-order inertial transfer function, with a FOPI controller considered. Sensitivity indices were constructed for the non-integer order operator described by the three most commonly used definitions: Grünwald–Letnikov, Riemann–Liouville, and Caputo.

It should be noted here that these indices can be applied to the analysis of any non-integer order model and control system, which decisively increases their scientific value.

#### **Discussion Points and Critical Remarks**

While reading the reviewed dissertation, some discussion-related remarks also came to mind. Some are of a more general nature, others are detailed. They are given below.

##### **4.1 General Remarks**

**(GR1)** Chapter 5 discusses and thoroughly analyzes integer-order models only for the flow process. An analogous analysis for the TRAS system model is definitely missing here.

**(GR2)** Chapter 3 introduces an object described by a non-integer order transfer function as a separately considered control plant, along with examples of systems that may be described by it. In the further part of the work, transfer functions of this type are used as substitute models for the two remaining, real objects.

##### **4.2 Detailed Remarks**

**(DR1)** In Chapter 3, the description of objects is inconsistent. From a mathematical point of view, the TRAS system model is described most precisely, but more technical details would be useful here. Preliminary example measurement data, as in the case of the flow system, is also missing. Conversely, from a hardware/measurement point of view, the system with the flow meter is described most precisely, but here, in turn, even the simplest mathematical model of the flow process is missing.

**(DR2)** p. 11, Equation (2.12) — in the GL definition, the sum is infinite. In the case of a finite sum, this is a PSE or FOB approximation.

**(DR3)** p. 17 (bottom) — it is worth adding that the Charef approximation describes a very specific class of fractional transfer functions in a form similar to the Strejc transfer function, but of fractional order. The formula for this transfer function should be given here.

**(DR4)** p. 46 — errors (4.3) and (4.4) could potentially be calculated analytically - were there such attempts?

**(DR5)** p. 47 — it is worth noting here that errors (4.8), (4.10) are calculated using the Final Value Theorem (FVT).

**(DR6)** p. 48, (4.12) — should this be capital K (static gain)?

**(DR7)** p. 50, Table 4.1 — what is a sense of testing errors for a CFE order of  $N=1$ ? There is no point in using such a low order. Much more could be learned from a test for  $N=3$ . Furthermore, it would be worth conducting similar tests for a low fractional order, e.g., 0.2 (0.5 and 0.8 are considered).

**(DR8)** p. 52 — it is stated that analytical formulas for the step responses of the considered inertial transfer functions are not known. This is not true, as appropriate formulas have been known for years and can be found in the literature (e.g., R. Caponetto, G. Dongola, L. Fortuna, I. Petráš, *Fractional Order Systems: Modeling and Control Applications*, World Scientific, 2010). In the considered case, they could be used as a reference for estimating approximation accuracy, e.g., using the IAE function. Consequently, Figures 4.1–4.5 cannot in any way be considered reliable regarding approximation accuracy.

**(DR9)** p. 64 onward, Table 4.6 does not present the solution of equation (4.21) but rather the results of its numerical minimization. The same applies to p. 75, Table 5.1 and equation (5.1).

**(DR10)** p. 71, Section 5.1 — discusses the identification of the integer-order model, while the model itself was not presented anywhere earlier. Models in the form of transfer functions (5.2) and (5.3) are given on the next page, but without justification of their form. The system's input and output, as well as the step response curves, were given much earlier, in Chapter 3, and appropriate cross-references should be included here.

**(DR11)** p. 75, Table 5.1 — model parameters are not obtained by solving equation (5.1) describing the MAE cost function, but by numerical minimization of this function.

**(DR12)** p. 92 — in the last sentence of Section 6.1.2, it is stated that the response is obtained by solving equation (6.8), but the analytical formula used for the solution and details of the numerical implementation (sampling period and memory length  $L$ ) are missing.

**(DR13)** p. 93 — formulas (6.12) and (6.13) are repeated. For what purpose?

**(DR14)** p. 93 — the delay model (6.14) was not identified in Chapter 5. What are its parameters?

**(DR15)** p. 96 — the steady-state error can be determined analytically, as shown in Chapter 4. For what purpose is it calculated numerically here?

**(DR16)** p. 117–118, Table 6.5 — in a typical situation, applying the GL definition for correctly modelling fractional derivatives requires an approximation order of at least 100. Why are very low orders, equal to  $m=2\ldots6$ , tested here?

**(DR17)** p. 119–121, Figs. 6.18–6.20 — experimental data curves should have been added for comparison here.

(DR18) p. 142, equation (8.3) — should the summation "start" from 1 instead of 0? Furthermore, what memory length  $M$  was used?

(DR19) p. 148, equation (8.14) — a fractional-order object with delay is considered here. It describes object (8.2) with  $M$  delays in state, but replacing it with a single-delay model must be explained more thoroughly. In a typical situation, a first-order model in the form (8.2) with  $M$  delays is replaced by an  $M$ -th order model without delay.

## **5 Formal Aspects**

The dissertation text was prepared in English. From a formal standpoint, the text does not raise major objections; linguistically, it is generally correct and "easy to read." The following editorial remarks can be formulated:

(E1) The table of contents is located after the acknowledgments, Polish and English abstracts, and the publication list. This does not facilitate reading.

(E2) In my opinion, the 3-level "nesting" of subsections is unnecessary; dividing the text into main chapters and subsections would be sufficient.

Apart from this, the work is prepared carefully from an editorial standpoint. During reading, a few minor language errors, typos, and punctuation errors were noticed, for example:

(L1) The first sentence of the Polish abstract reads (translated back): "Real-time systems require regulated operation to ensure a given quality of performance, which is usually implemented via feedback controllers." The content of this sentence clearly requires thorough correction. It was likely written by an AI tool used for automatic translation, but in such a situation, the translation should always be checked and corrected if necessary.

(L2) p. 3 — Objective 1: it is not clear what the influence of what on what is being analyzed. One can guess it concerns the influence of the choice of approximation method on model accuracy, but this is completely unclear.

(L3) p. 197 — "modeling" is repeated.

(L4) p. 201 — better: "fractional."

(L5) p. 232, l. 4 — "Python."

(L6) p. 241, l. 6 — should be "determines."

(L7) p. 242 — should be "related."

(L8) p. 431, l. 3,14 — should be "presents," "describes."

(L9) p. 51, l. 4 — should be: "...using approximated models..."

(L10) p. 69, l. 8 — should be: "addresses."

(L11) p. 71 — title of Section 5.1: should be "...reduced order."

(L12) p. 76, l. 2 — should be "isolate."

(L13) p. 85, l. 5 — should be "shows."

## **6 Summary and Conclusion**

Based on the review of the dissertation's content, I conclude that its Author possesses **deep theoretical knowledge** in the fields of metrology, control systems, dynamic system modelling, and non-integer order differential calculus.

He also demonstrated **creativity and the ability to critically analyze** both existing and his own solutions, indicating the ability to independently conduct scientific work. The dissertation presents an **original solution** to the posed scientific problem, which is the development of a new, original method for analyzing the uncertainty of non-integer order models and control systems. The obtained results constitute a **significant contribution** to the field of applications of non-integer order calculus in automatic control.

At the same time, I firmly state that the discussion points and critical remarks given in sections 4 and 5 do **not diminish the value** of the reviewed dissertation.

## **✓ Final Conclusion**

In summary, in my opinion, the reviewed dissertation meets the requirements set for doctoral dissertations, in accordance with the Act of 20 July 2018 on Higher Education and Science (Journal of Laws of 2022, item 574, as amended), in the field of Engineering and Technical Sciences, in the discipline of Automatic Control, Electronics, Electrical Engineering and Space Technologies.

I therefore **request the Honorable Committee**, appointed by the Council of the Discipline of Automatic Control, Electronics, Electrical Engineering and Space Technologies of the Silesian University of Technology in Gliwice, to **accept the dissertation** and admit its Author, **Faisal Saleem, M.Sc. Eng., to the public defense.**

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